

## nag\_arma\_time\_series (g05hac)

### 1. Purpose

**nag\_arma\_time\_series (g05hac)** generates an autoregressive moving average (ARMA) time series with normally distributed errors (or residuals). It initialises the series to a stationary position and sets up a reference vector enabling the function to be called repeatedly, adding terms to the previous series at each call.

### 2. Specification

```
#include <nag.h>
#include <nagg05.h>

void nag_arma_time_series(Boolean start, Integer p, Integer q,
                          double phi[], double theta[], double mean, double vara,
                          Integer n, double w[], double ref[], NagError *fail)
```

### 3. Description

An ARMA model, denoted by  $\text{ARMA}(p, q)$ , is a mixture of an autoregressive process of order  $p$  (AR) and a moving average (MA) process of order  $q$  and can be written as

$$(x_n - \mu) = \phi_1(x_{n-1} - \mu) + \dots + \phi_p(x_{n-p} - \mu) + a_n - \theta_1 a_{n-1} \dots - \theta_q a_{n-q}$$

where  $x_n$  are the realization of the series,  $\mu$  is the mean of the series and  $a_n$  are the errors (or residuals, also often called the white noise) which are independently distributed as normal with mean zero and variance  $\sigma^2$ . The parameters  $\phi_i$  are the autoregressive parameters and the parameters  $\theta_i$  are the moving average parameters.

The function sets up initial values corresponding to a stationary position using the method described by Tunnicliffe-Wilson (1979). It generates  $n$  terms of the time series by first calculating the next term in the autoregressive series and then applying the moving-average summation and storing the result.

### 4. Parameters

#### start

Input: **start** must be **TRUE** if a new series is to begin, if **start** is **FALSE** a previously generated series will be continued. If **start** is **FALSE** then the scalar parameters **p**, **q**, **mean** and **vara** and the contents of the array parameters, **phi** and **theta** must not be changed.

#### p

Input: the number of autoregressive coefficients supplied.  
Constraint:  $p \geq 0$ .

#### q

Input: the number of moving-average coefficients supplied.  
Constraint:  $q \geq 0$ .

#### phi[p]

Input: the autoregressive coefficients of the model, if any, **phi**[ $i - 1$ ] must contain  $\phi_i$  for  $i = 1, 2, \dots, p$ .

#### theta[q]

Input: the moving-average coefficients of the model, if any, **theta**[ $i - 1$ ] must contain  $\theta_i$  for  $i = 1, 2, \dots, q$ .

#### mean

Input: the mean of the time series.

#### vara

Input: the variance of the errors,  $\sigma^2$ .  
Constraint: **vara** > 0.0.

- n**  
Input: the number of observations to be generated.  
Constraint:  $n \geq 1$ .
- w[n]**  
Output: the realization of the time series.
- ref [5\*MAX(p,q)+7]**  
Output: the reference vector and the recent history of the series.
- fail**  
The NAG error parameter, see the Essential Introduction to the NAG C Library.

## 5. Error Indications and Warnings

### NE\_INT\_ARG\_LT

- On entry, **p** must not be less than 0: **p** = *<value>*.
- On entry, **q** must not be less than 0: **q** = *<value>*.
- On entry, **n** must not be less than 1: **n** = *<value>*.

### NE\_REAL\_ARG\_LE

- On entry, **vara** must not be less than or equal to 0.0: **vara** = *<value>*.

### NE\_STATIONARITY

- The input series does not constitute a stationary time-series model.

### NE\_START\_P\_Q

- The function has been called either with **start** = **FALSE** the first time or at least one of **p** or **q** has been changed in a subsequent call with **start** = **FALSE**.

### NE\_REF\_VEC

- The reference vector set up by the previous call of this function has become corrupt.

## 6. Further Comments

None.

### 6.1. References

- Knuth D E (1981) *The Art of Computer Programming (Vol 2)*. (2nd Edn) Addison-Wesley.
- Tunncliffe Wilson G (1979) Some Efficient Computational Procedures for High Order ARMA Models. *J. Stat. Comput. Simul.* **8** 301–309.

## 7. See Also

None.

## 8. Example

The program below shows two calls of nag\_arma\_time\_series. In the first call an ARMA series is generated. In the second call terms are added to the already existing series.

### 8.1. Program Text

```
/* nag_arma_time_series(g05hac) Example Program
 *
 * Copyright 1994 Numerical Algorithms Group.
 *
 * Mark 3, 1994.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg05.h>
```

```
#define NA 3
#define NB 2
#define NR 20
#define NW 10

main()
{
    Integer i, ip, iq, n;
    double phi[NA], theta[NB], w[NW];
    double mean, vara;
    double ref[NR];
    Boolean start;
    Integer seed = 0;

    Vprintf("g05hac Example Program Results\n\n");

    g05cbc(seed);
    ip = 2;
    iq = 0;
    n = NW;
    phi[0] = 0.4;
    phi[1] = 0.2;
    mean = 0.0;
    vara = 2.0;

    /* Generate an ARMA series with 5 terms */
    start = TRUE;
    g05hac(start, ip, iq, phi, theta, mean, vara, (Integer)5, w, ref,
          NAGERR_DEFAULT);

    /* Add further 5 terms to the previous series*/
    start = FALSE;
    g05hac(start, ip, iq, phi, theta, mean, vara, (Integer)5, &w[5], ref,
          NAGERR_DEFAULT);
    for (i = 0; i < n; ++i)
        Vprintf("%12.4f \n", w[i]);
    exit(EXIT_SUCCESS);
}
```

## 8.2. Program Data

None.

## 8.3. Program Results

g05hac Example Program Results

```
3.4060
1.6952
3.5042
1.1311
0.0640
0.5834
0.5352
-1.7206
-0.4964
1.6448
```

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